NORTHERN BEACHES SECONDARY COLLEGE

MANLY SELECTIVE CAMPUS

Year 12

Trial Examination

2022

Mathematics Extension II

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Write your name on the front of every booklet.
- In Questions 11 to 16 show relevant mathematical reasoning and/or calculations.
- NESA approved calculators and templates may be used.
- Weighting: 30%

Section I Multiple Choice

- 10 marks
- Attempt all questions.
- Answer Sheet provided
- Allow about 15 minutes for this section

Section II Free Response

- 90 marks
- Start a separate booklet for each question.
- Each question is of equal value.
- All necessary working should be shown in every question.
- Allow about 2 hours and 45 minutes for this section.

Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

Q1. Given that $1 - i$ is a solution of $z^3 - 4z^2 + 6z - 4 = 0$, what are the other solutions?

A. $1 + i$ and -6

B. $1 + i$ and 2

C. $-1 - i$ and $4 + 2i$

D. $1 + i$ and -4

Q2. Let $\underline{u} = 2\underline{i} - a\underline{j} - \underline{k}$ and $\underline{v} = 3\underline{i} + 2\underline{j} - b\underline{k}$.
If these vectors are perpendicular to each other, what are the possible values for a and b?

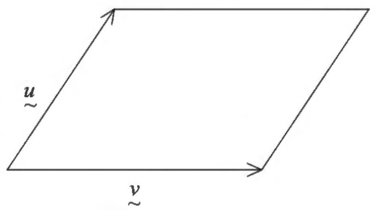
A. $a = 2$ and $b = 2$

B. $a = -2$ and $b = 10$

C. $a = 0$ and $b = 0$

D. $a = -1$ and $b = -8$

Q3. The diagram below shows a rhombus.



Adjacent sides are represented by two vectors, $\underset{\sim}{u}$ and $\underset{\sim}{v}$.

It follows that:

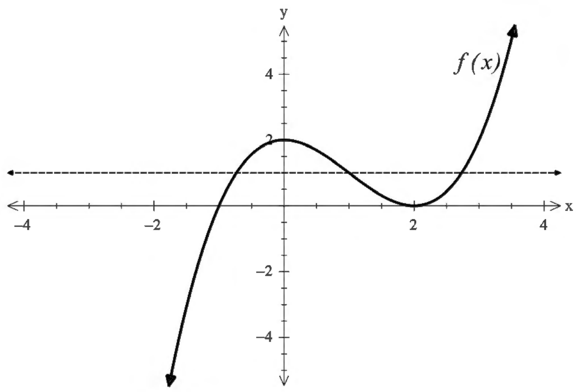
A. $\underset{\sim}{u} \cdot \underset{\sim}{v} = 0$

B. $\underset{\sim}{u} = \underset{\sim}{v}$

C. $(\underset{\sim}{u} + \underset{\sim}{v}) \cdot (\underset{\sim}{u} - \underset{\sim}{v}) = 0$

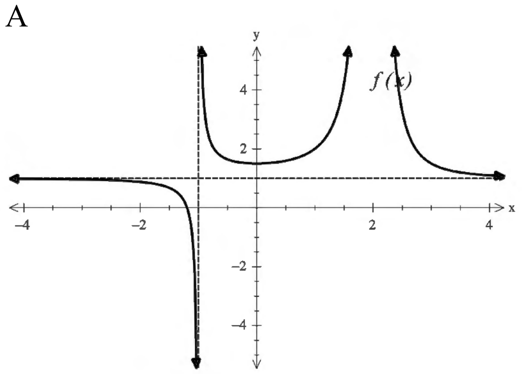
D. $|\underset{\sim}{u} + \underset{\sim}{v}| = |\underset{\sim}{u} - \underset{\sim}{v}|$

Q4. The graph of the function $y = f(x)$ is shown below.

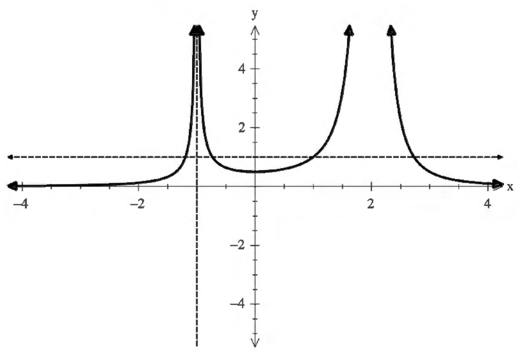


Which graph below shows the most accurate representation of the graph $y = \dfrac{1}{f(x)}$?

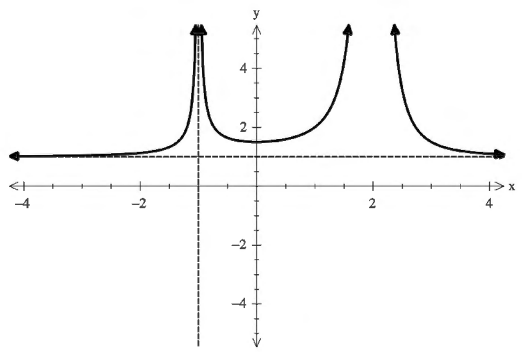
A



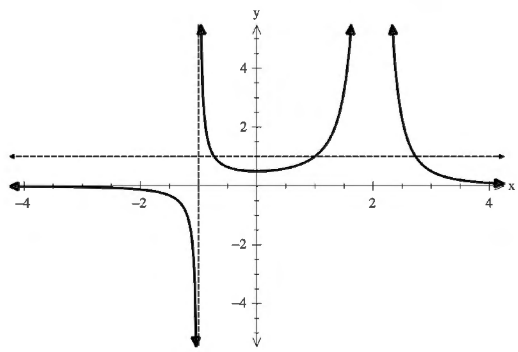
B



C



D



Q5. Consider the statement:

If the soup contains lemon or vinegar, the soup does not contain chilli.

Which of the following is logically equivalent to the statement?

A. If the soup contains chilli, the soup does not contain lemon nor vinegar.

B. If the soup contains chilli, the soup does not contain lemon or the soup does not contain vinegar.

C. If the soup does not contain lemon or the soup does not contain vinegar, the soup contains chilli.

D. If the soup does not contain lemon nor vinegar, the soup contains chilli.

Q6. The vector equation of the line that passes through the points A (1, 0, 2) and B (3, 9, 6) is given by which of the following?

A. $\underset{\sim}{r} = \underset{\sim}{i} + 2\underset{\sim}{k} + \lambda\left(2\underset{\sim}{i} + 9\underset{\sim}{j} + 4\underset{\sim}{k}\right)$

B. $\underset{\sim}{r} = \underset{\sim}{i} + 2\underset{\sim}{k} + \lambda\left(2\underset{\sim}{i} + 9\underset{\sim}{j} + 8\underset{\sim}{k}\right)$

C. $\underset{\sim}{r} = \underset{\sim}{i} + 2\underset{\sim}{k} + \lambda\left(4\underset{\sim}{i} + 9\underset{\sim}{j} + 4\underset{\sim}{k}\right)$

D. $\underset{\sim}{r} = \underset{\sim}{i} + 2\underset{\sim}{k} + \lambda(2\underset{\sim}{i} + 4\underset{\sim}{k})$

Q7. Which expression is equal to $\int \tan^{-1} x \, dx$?

A. $x\tan^{-1} x - \int \frac{x}{1+x^2}\,dx$

B. $x^2\tan^{-1} x - \int \frac{x^2}{1+x^2}\,dx$

C. $\frac{1}{2}\left(\tan^{-1} x\right)^2$

D. $\frac{x}{2}\left(\tan^{-1} x\right)^2 - \int \frac{x}{\tan x}\,dx$

Q8. Which of the following statements is FALSE?

A. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ such that $x^4 = 2y^3$

B. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ such that $\mathrm{x} = \mathrm{y} \Leftrightarrow \sin x = \sin y$

C. $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}$ such that $\sqrt{x} < e^y$

D. $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}$ such that $|y| > x$

Q9. $\int_{-1}^{2} 2x\sqrt{x+2}\,dx$.

A. $\frac{46}{15}$

B. $\frac{92}{15}$

C. $\frac{113}{2}$

D. $\frac{28+32\sqrt{2}}{15}$

Q10. If $\int_a^b \frac{1}{\sin x + \cos x}\,dx = 1$, where $a, b \in \mathbb{R}$,

what is the value of $\int_{2a}^{2b} \frac{1}{\sin\frac{x}{2} + \cos\frac{x}{2}}\,dx$

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 1

D. 2

End of Multiple Choice

Section II

90 marks

Attempt Questions 11 – 16

Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 **15 marks**

a. Let $z = 4cis\frac{5\pi}{6}$ and $\omega = \sqrt{2}cis\frac{\pi}{4}$.

(i) Find z^7 in modulus-argument form. **2**

(ii) Express $\frac{z}{w}$ in both modulus-argument form and Cartesian form. **3**

(iii) Hence or otherwise, find the exact value of $\cos\frac{7\pi}{12}$. **2**

b. A point P moves on the $x-$axis so that its $x-$coordinate, in metres, at time t seconds is given by

$$x = 8[1 - \sin(2t)] + 3\cos(2t)$$

(i) Show that the motion of P is simple harmonic. **2**

(ii) State centre and period of the motion of P. **2**

c. Sketch the region on the Argand diagram defined by the following

$$\{\, z: 1 < |z| \le 3\} \cap \{\, z: Im(z) > Re(z)\}$$ **4**

End of Question 11

Question 12 **15 marks**

a. (i) Find the values of a, b and c such that **3**

$$\frac{5x}{(x-1)(x^2+2x+2)} \equiv \frac{a}{x-1} + \frac{bx+c}{x^2+2x+2}$$

(ii) Hence, or otherwise, find **3**

$$\int \frac{5x}{(x-1)(x^2+2x+2)} dx.$$

b. Let $\omega = e^{i(\pi/n)}$, where ϖ is a complex number ie. ω *is not* $\in \mathbb{R}$

(i) Show that ω is a solution of $z^{2n} = 1$. **1**

(ii) Given $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + \ldots + b^{n-1})$,
deduce that $1 + \omega + \cdots + \omega^{2n-1} = 0$. **1**

(iii) Show that $1 + \omega + \cdots + \omega^{n-1} = 1 + i\cot\left(\frac{\pi}{2n}\right)$. **3**

c. Prove the following statement by contradiction. **4**

If a, b and c are odd integers and $x \in \mathbb{Z}$, then the equation $ax^2 + bx + c = 0$ has no integer solution.

End of Question 12

Question 13 **15 marks**

a. (i) Express $1 + i\cot\theta$ in modulus-argument form, where $0 < \theta < \frac{\pi}{2}$ **2**

(ii) Hence, or otherwise, simplify $(1 + i\cot\theta)^{-3}$ **1**

b. Find the exact value of $\displaystyle\int_0^{\frac{\pi}{2}} \frac{1}{2 - \sin x}\, dx$. **4**

c. A sequence is defined recursively as $u_1 = 1,\ u_{n+1} = 3u_n + 2n + 1$, for all positive integers n.
]
Use Mathematical Induction to prove that $u_n = 3^n - n - 1$ for all positive integers n. **3**

d. A particle is initially at rest on the number line at a position of $x = 1$. The particle moves continuously along the number line according to the acceleration equation

$$\ddot{x} = \frac{4}{(x-2)^2} + \frac{8}{x^3}.$$

At time t, the velocity of the particle is v.

(i) Show that $v^2 = -\frac{8}{x-2} - \frac{8}{x^2}$ **2**

(ii) Hence, or otherwise, determine the possible range of particle's displacement as it moves along the number line. **3**

Question 13 completed.

Question 14 **15 Marks**

a. Use an appropriate substitution to evaluate $\int_0^4 \frac{\sqrt{x}}{(x\sqrt{x}+1)^2}\,dx$ **3**

b. Consider the statement about two positive integers *m* and *n*.
"If m^2+n^2 is even, then either both *m* and *n* are even or both *m* and *n* are odd."

(i) Write down the contrapositive of the statement. **1**

(ii) Prove the original statement by contraposition. **2**

c. (i) If $0<a<4$, prove that $\frac{1}{a}+\frac{1}{4-a}\geq 1$. **2**

(ii) If $0<a<4, 0<b<4$ and $0<c<4$, prove that at least one of the numbers

$$\frac{1}{a}+\frac{1}{4-b},\ \frac{1}{b}+\frac{1}{4-c},\ \frac{1}{c}+\frac{1}{4-a}$$

is greater than or equal to one. **3**

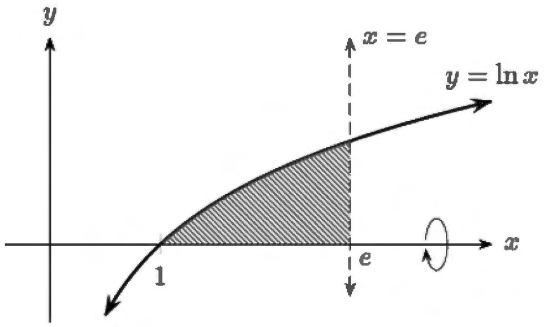
d. Let $I_n = \int_0^1 \frac{x^{2n}}{\sqrt{1+x^2}}\,dx$, where $n = 1, 2, 3, \ldots$. **4**

Deduce that $2nI_n + (2n-1)I_{n-1} = \sqrt{2}$, where $n = 2, 3, 4, \ldots$.

Question 14 completed.

Question 15 **15 marks**

a. The region enclosed by the curve $y = \ln x$, the line $x = e$ and the x axis is shown in the diagram. **3**



Find the volume of the solid formed by rotating the shaded region about the x axis.

b. Sphere S has a vector equation

$$\left| \underset{\sim}{r} - \left(3\underset{\sim}{i} + \underset{\sim}{j} + 4\underset{\sim}{k}\right) \right| = \sqrt{35}$$

(i) Write the Cartesian equation of this sphere. **1**

(ii) The line l has the equation $\underset{\sim}{r} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \lambda \in \mathbb{R}$. **3**

Determine whether this line is a tangent to the sphere S or not. Give full reasons for your answer.

c. Show there exists complex numbers z and w, such that **3**

$\frac{1}{z} + \frac{1}{w} = \frac{1}{z+w}$ where $z \neq 0$ and $w \neq 0$.

d. (i) Show $a^2 + b^2 + c^2 \geq ab + ac + bc$ **1**

(ii) Hence, show $(a+b+c)^2 \geq 3\ (ab + ac + bc)$ **1**

(iii) Hence, prove $a^2 + b^2 + c^2 \geq 3 \times \sqrt[3]{a^2 b^2 c^2}$ **3**

(Given $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Question 15 completed

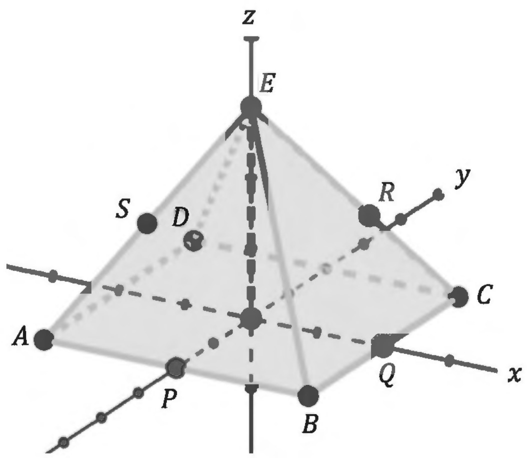
Question 16 **15 marks**

a. (i) Solve $z^3 = -1$, giving your answers in modulus-argument form. **2**

(ii) Using part (i), or otherwise, find the solutions to the equation **3**

$$(z^2 + 2)^3 = -1$$

b. ABCDE is a pyramid with base measuring 8 cm by 8 cm.
The height of the pyramid is $4e$, where e is a real positive number.
The origin O is the centre of the base.



The points P, Q, R and S are the midpoints of sides AB, BC, CE and EA respectively.

The position vectors of A , B, C and D relative to O, respectively, are

$$-4\underset{\sim}{i} - 4\underset{\sim}{j}\ ,\ 4\underset{\sim}{i} - 4\underset{\sim}{j}\ ,\ 4\underset{\sim}{i} + 4\underset{\sim}{j} \text{ and } -4\underset{\sim}{i} + 4\underset{\sim}{j}.$$

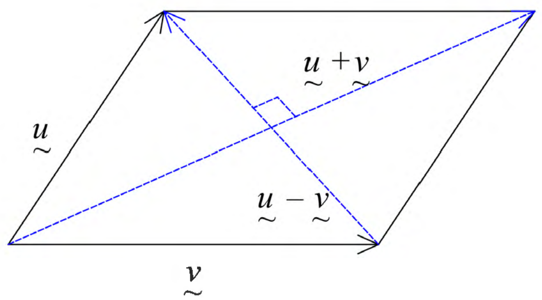
(i) Show, in terms of e, that $\overrightarrow{PR} = 2\underset{\sim}{i} + 6\underset{\sim}{j} + 2e\underset{\sim}{k}$ and $\overrightarrow{QS} = -6\underset{\sim}{i} - 2\underset{\sim}{j} + 2e\underset{\sim}{k}$. **3**

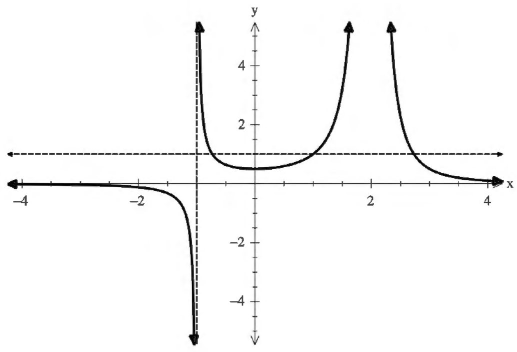
(ii) Hence or otherwise, write the vector equations of line l, passing through points P and R, and line s, passing through points Q and S. **2**

(iii) Find the position vector $\overrightarrow{OM}$ in terms of e, where M is the point of intersection of lines l and s. **2**

(iv) Given that $\overrightarrow{OM}$ is perpendicular to $\overrightarrow{EB}$, find the value of e and hence find the acute angle between $\overrightarrow{PR}$ and $\overrightarrow{QS}$. Give your answer to the nearest degree. **3**

End of paper

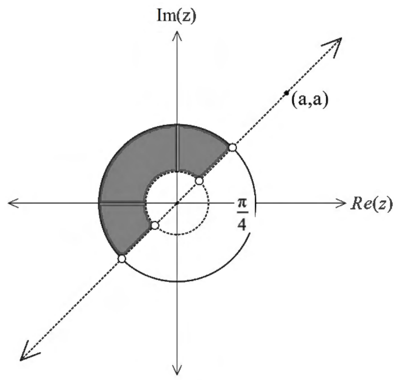
Q1	$z^3 - 4z^2 + 6z - 4 = 0$ roots are $(1 - i)$ and $(1 + i)$ factor $z^2 - 2Re(\alpha)z + \lvert\alpha\rvert^2$ $\therefore\ z^2 - 2z + 2$ $z^3 - 4z^2 + 6z - 4 = (z^2 - 2z + 2)(az + b)$ $\therefore\quad a = 1$ $b = -2$ Other roots are $1 + i$ and 2	B
Q2	$u = 2i - aj - k$ $v = 3i + 2j - bk$ $u \cdot v = 0$ $u \cdot v = 2 \times 3 - 2a + b = 0$ $b - 2a = -6$ $-8 - 2(-1) = -6$ $a = -1 \qquad b = -8$	D
Q3	 Diagonals of rhombus are perpendicular therefore $(\underset{\sim}{u} + \underset{\sim}{v}) \cdot (\underset{\sim}{u} - \underset{\sim}{v}) = 0$	C

Q4		D
Q5	**If the soup contains chilli, the soup does not contain lemon nor vinegar.**	A
Q6	$A = \underset{\sim}{i} + 2\underset{\sim}{k}$ $B = 3\underset{\sim}{i} + 9\underset{\sim}{j} + 6\underset{\sim}{k}$ $\overrightarrow{\text{AB}} = 2\underset{\sim}{i} + 9\underset{\sim}{j} + 4\underset{\sim}{k}$ $\underset{\sim}{r} = \underset{\sim}{i} + 2\underset{\sim}{k} + \lambda\left(2\underset{\sim}{i} + 9\underset{\sim}{j} + 4\underset{\sim}{k}\right)$	A
Q7	$u = \tan^{-1} x \qquad v' = 1$ $u' = \frac{1}{1+x^2} \qquad v = x$ $I = uv - \int \text{vu}'\, dx$ $= x\tan^{-1} x - \int \frac{x}{1+x^2}\, dx$	A
Q8	$\forall \boldsymbol{x} \in \mathbb{R}, \exists \boldsymbol{y} \in \mathbb{R}$ **such that** $\mathbf{x = y} \Leftrightarrow \mathbf{sin}\, \boldsymbol{x} = \mathbf{sin}\, \boldsymbol{y}$	B

Q9	$\int_{-1}^{2} 2x\sqrt{x+2}\, dx$ $l\eta\ u = x+2$ $x = u-2 \Rightarrow dx = du$ $x = -1 \Rightarrow u = 1$ $x = 2 \Rightarrow u = 4$ $\int_{1}^{4} 2(u-2)\sqrt{u}\, du$ $= 2\int_{1}^{4} u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\, du$ $= 2\left[\frac{2}{5}u^{\frac{5}{2}} - \frac{4}{3}u^{\frac{3}{2}}\right]_{1}^{4}$ $= 2\left\{\left(\frac{2}{5}\times 2^{5} - \frac{4}{3}\times 2^{3}\right) - \left(\frac{2}{5}\times 1 - \frac{4}{3}\times 1\right)\right\}$ $= 2\left(\frac{64}{5} - \frac{32}{3} - \frac{2}{5} + \frac{4}{3}\right) = \frac{92}{15}$	A
Q10	$\int_{a}^{b} \frac{1}{\sin x + \cos x}\, dx = 1$ $\text{let } x = \frac{u}{2} \Rightarrow 2x = u$ $2dx = du$ $x = a \Rightarrow u = 2a$ $x = b \Rightarrow u = 2b$ $\therefore \int_{2a}^{2b} \frac{2}{\sin\frac{u}{2} + \cos\frac{u}{2}}\, du = 1$ $\int_{2a}^{2b} \frac{1}{\sin\frac{x}{2} + \cos\frac{x}{2}}\, dx = \frac{1}{2}$	B

Question 11.

a-i	$z = R\text{cis}\theta$ $z^7 = R^7\text{cis}(7\theta)$ $\therefore z^7 = 4^7\text{cis}\frac{35\pi}{6}$ $= 16384\text{cis}\left(-\frac{\pi}{6}\right)$	2 marks – correct solution 1 mark – either - correct argument - correct modulus
a-ii	ModArg form $\frac{z_1}{z_2} = \frac{R_1\text{cis}\theta}{R_2\text{cis}\beta} = \frac{R_1}{R_2}\text{cis}(\theta - \beta)$ $= \frac{4}{\sqrt{2}}\text{cis}\left(\frac{5\pi}{6} - \frac{\pi}{4}\right)$ $= 2\sqrt{2}\text{cis}\frac{7\pi}{12}$ Cartesian form: $\frac{z}{w} = \frac{-2\sqrt{3}+2i}{1+i} \times \frac{1-i}{1-i} = (1-\sqrt{3}) + i(1+\sqrt{3})$ $\frac{z}{w} = \frac{-2\sqrt{3}+2}{2} + \frac{2+2\sqrt{3}}{2}i$ was also accepted Decimal Form was *NOT* accepted	3 marks Provides correct solution 2 marks Expresses fraction in both modulus-argument or Cartesian form (but with the denominator not realised), or equivalent merit 1 mark - uses $\frac{z_1}{z_2}$ to produce correct modulus or argument - uses $\frac{z}{\omega}$ to produce correct fraction
a-iii	Using part (ii) and equating the real parts, $2\sqrt{2}\cos\frac{7\pi}{12} = 1 - \sqrt{3}$ $\therefore \cos\frac{7\pi}{12} = \frac{1-\sqrt{3}}{2\sqrt{2}}$	2 marks – correct solution 1 mark - Attempts to equate real or imaginary parts of the two expression in part (i), or equivalent merit

c-i	Note: $x = 8[1 - \sin(2t)] + 3\cos(2t)$ $\therefore x - 8 = -8\sin(2t) + 3\cos(2t)$ $\dot{x} = -16\cos(2t) - 6\sin(2t)$ $\ddot{x} = 32\sin(2t) - 12\cos(2t)$ $= -4(x - 8)$ which is in the form $-n^2(x - c)$ Hence motion is SHM	2 marks – correct solution 1 mark – correct approach with one arithmetic error.
c-ii	Centre $x = 8$, period$=\dfrac{2\pi}{2} = \pi$	2 marks – both answers 1 mark – one answer only
d	 Features to include 1. Correct circles and shading. 2. Correct boundaries ie. full or dotted line 3. Correct angle or indicated line is $y = x$ or equivalent, 4. Open circles used correctly.	4 marks – one mark for each correct feature

Question 12

a-i	$5x \equiv a(x^2+2x+2)+(x-1)(bx+c)$ Let $x=1$: $5=5a \Rightarrow a=1$ $x=0$: $0=2a-c \Rightarrow c=2$ $x=-1$: $-5=a-2(-b+c) \Rightarrow b=-1$	3marks – correct solution 2 marks – correct approach with one arithmetic error. 1 mark – correct initial equation equating numerators
a-ii	$I=\int \frac{5x}{(x-1)(x^2+2x+2)}dx=\int\left(\frac{1}{x-1}+\frac{-x+2}{x^2+2x+2}\right)dx$ $I=\int\left(\frac{1}{x-1}-\frac{\frac{1}{2}(2x+2)-3}{x^2+2x+2}\right)dx$ $I=\int\left(\frac{1}{x-1}-\frac{1}{2}\times\frac{2x+2}{x^2+2x+2}+\frac{3}{(x+1)^2+1}\right)dx$ $I=\ln\lvert x-1\rvert-\frac{1}{2}\ln(x^2+2x+2)+3\tan^{-1}(x+1)+c$ or $I=\ln\left\lvert\frac{x-1}{\sqrt{x^2+2x+2}}\right\rvert+3\tan^{-1}(x+1)+c$	3 marks – correct solution 2 marks – - writes the integrand as the correct sum of fractions and completes the square in a denominator, or equivalent merit 1 mark - correct integration achieving $\ln\lvert x-1\rvert$ - attempts to write as sum of three fractions.
b-i	$\omega^{2n}=\left(e^{\pi i/n}\right)^{2n}$ $=e^{2\pi i}$ $=1$	1 mark - Provides correct solution
b-ii	$\omega^{2n}-1=0$ $\Rightarrow(\omega-1)\left(\omega^{2n-1}+\cdots+\omega+1\right)=0$ Since $\omega\neq1$, $\omega^{2n-1}+\cdots+\omega+1=0$	1 mark - Provides correct solution including statement that $\omega \neq 1$

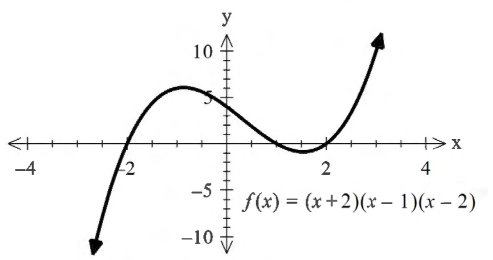
b-iii	$$1+\omega+\cdots+\omega^{n-1}=\frac{1-\omega^n}{1-\omega}$$ $$=\frac{1-e^{\pi i}}{1-\cos\left(\frac{\pi}{n}\right)-i\sin\left(\frac{\pi}{n}\right)}$$ Line 1 $$=\frac{1-(-1)}{1-\cos\left(\frac{\pi}{n}\right)-i\sin\left(\frac{\pi}{n}\right)}\times\frac{1-\cos\left(\frac{\pi}{n}\right)+i\sin\left(\frac{\pi}{n}\right)}{1-\cos\left(\frac{\pi}{n}\right)+i\sin\left(\frac{\pi}{n}\right)}$$ $$=\frac{2\left(1-\cos\left(\frac{\pi}{n}\right)+i\sin\left(\frac{\pi}{n}\right)\right)}{\left(1-\cos\left(\frac{\pi}{n}\right)\right)^2+\left(\sin\left(\frac{\pi}{n}\right)\right)^2}$$ Line 2 $$=\frac{2\left(1-\cos\left(\frac{\pi}{n}\right)+i\sin\left(\frac{\pi}{n}\right)\right)}{2-2\cos\left(\frac{\pi}{n}\right)}$$ $$=\frac{1-\cos\left(\frac{\pi}{n}\right)}{1-\cos\left(\frac{\pi}{n}\right)}+i\frac{\sin\left(\frac{\pi}{n}\right)}{1-\cos\left(\frac{\pi}{n}\right)}$$ $$=1+i\frac{2\sin\left(\frac{\pi}{2n}\right)\cos\left(\frac{\pi}{2n}\right)}{2\sin^2\left(\frac{\pi}{2n}\right)}$$ $$=1+i\cot\left(\frac{\pi}{2n}\right)$$	3 marks - Provides correct solution 2 marks - correct to Line 2 1 mark – Produces either fraction in Line 1.

c	Suppose for a contradiction that a, b and c are odd integers and the equation $ax^2 + bx + c = 0$ has an integer solution. For $A, B, C \in \mathbb{Z}$, let $a = 2A + 1$, $b = 2B + 1$, and $c = 2C + 1$ $\therefore\ (2A+1)x^2 + (2B+1)x + (2C+1) = 0$ [*] **Case 1:** x is an even number, i.e. $x = 2k$, $k \in \mathbb{Z}$. Since x is a solution, substitute into [*]: $(2A+1)(2k)^2 + (2B+1)(2k) + (2C+1) = 0$ Expand to give: $2[2k^2(2A+1) + k(2B+1) + C] + 1 = 0$ LHS is an odd number which cannot equal to zero, hence absurd. **Case 2:** x is an odd number, i.e. $x = 2k + 1$, $k \in \mathbb{Z}$. Since x is a solution, substitute into [*]: $(2A+1)(2k+1)^2 + (2B+1)(2k+1) + (2C+1) = 0$ Expand to give: $(2A+1)(4k^2+4k+1) + 4Bk + 2B + 2k + 1 + 2C + 1 = 0$ $2A(4k^2+4k) + 4k^2 + 6k + 3 + 4Bk + 2B + 2C = 0$ $2[A(4k^2+4k) + 2k^2 + 3k + 1 + 2Bk + B + C] + 1 = 0$ LHS is an odd number which cannot equal to zero, hence absurd. $\therefore$ initial supposition is false $\therefore$ if a, b and c are odd integers, then $ax^2 + bx + c = 0$ has no integer solution	4 marks – correct solution. 3 marks – attempts both odd and even expressions but only one correct. 2 marks - attempts one case only and proven correctly. - attempts to use quadratic formula to to show no integer solutions ie. more than simply deriving quad formula 1 mark – correctly converts a, b and c to an expression for odd numbers.

Question 13

a-i	$1 + i\cot\theta \quad = 1 + i\frac{\cos\theta}{\sin\theta}$ $= \frac{\sin\theta + i\cos\theta}{\sin\theta}$ $= \frac{1}{\sin\theta}[\sin\theta + i\cos\theta]$ $= \frac{1}{\sin\theta}\left[\cos\left(\frac{\pi}{2} - \theta\right) + i\sin\left(\frac{\pi}{2} - \theta\right)\right]$ $= cosec\ \theta\ cis\ \left(\frac{\pi}{2} - \theta\right)$ $\sqrt{1 + \cot\theta^2}$ was also accepted. If the argument was in degrees this was also accepted.	2 marks – correct solution 1 mark – either - correct modulus - correct argument
a-ii	$(1 + i\cot\theta)^{-3} \quad = \left(cosec\ \theta\ cis\ \left(\frac{\pi}{2} - \theta\right)\right)^{-3}$ $= cosec^{-3}\ \theta\ cis\ \left[-3\left(\frac{\pi}{2} - \theta\right)\right]$ $= \sin^3\theta\ \ cis\left(3\theta - \frac{3\pi}{2}\right)$ $= \sin^3\theta\ \ cis\left(3\theta + \frac{\pi}{2}\right)$	1 mark
b	Let $t = \tan\frac{x}{2} \quad dt = \frac{1}{2}\sec^2\frac{x}{2}dx = \frac{1}{2}(1 + t^2)dx \quad dx = \frac{2}{1+t^2}dt$ $\therefore \sin x = \frac{2t}{1 + t^2}$ When $x = 0, t = 0$ $x = \frac{\pi}{2}, t = 1$ $\therefore I = \int_0^{\frac{\pi}{2}} \frac{1}{2 - sinx}dx = \int_0^1 \frac{1}{2 - \frac{2t}{1 + t^2}} \frac{2}{1 + t^2}\ dt$ $I = \int_0^1 \frac{2}{2 - \frac{2t}{1 + t^2}} \times \frac{2}{1 + t^2}\ dt$ $I = \int_0^1 \frac{2}{2(1 + t^2) - 2t}\ dt$ $I = \int_0^1 \frac{2}{2t^2 - 2t + 2}\ dt$	4 marks - Provides correct solution 3 marks - Finds the correct integral, or equivalent merit 2 marks - Completes t-substitution, including the limits of integration, or equivalent merit 1 mark - Attempts to use a t-substitution, or equivalent merit

	$= \int_0^1 \frac{1}{t^2 - t + 1}\,\mathrm{dt}$ $= \int_0^1 \frac{1}{\left(t - \frac{1}{2}\right)^2 + \frac{3}{4}}\,\mathrm{dt}$ $= \left[\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2t-1}{\sqrt{3}}\right)\right]_0^1$ $= \frac{2}{\sqrt{3}}\tan^{-1}\frac{1}{\sqrt{3}} - \frac{2}{\sqrt{3}}\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ $= \frac{2}{\sqrt{3}} \times \frac{\pi}{6} - \frac{2}{\sqrt{3}} \times -\frac{\pi}{6}$ $= \frac{4\pi}{6\sqrt{3}}$ $= \frac{2\pi}{3\sqrt{3}}$ $= \frac{2\sqrt{3}\pi}{9}$	

c	$P(n): u_n = 3^n - n - 1, n \in Z^+$, for the sequence $u_1 = 1$, $u_{n+1} = 3u_n + 2n + 1, n \in Z^+$. Prove $P(1)$ true: By formula, $u_1 = 3^1 - 1 - 1 = 1$ By definition, $u_1 = 1$ $\therefore P(1)$ is true Assume $P(k)$ true, $k \in \mathbb{Z}^+$ i.e. $u_k = 3^k - k - 1$ Prove $P(k+1)$ true. i.e. prove $u_{k+1} = 3^{k+1} - (k+1) - 1 = 3^{k+1} - k - 2$ Now, from the definition: $u_{k+1} = 3u_k + 2k + 1$ $u_{k+1} = 3(3^k - k - 1) + 2k + 1$ using the assumption $u_{k+1} = 3^{k+1} - 3k - 3 + 2k + 1$ $u_{k+1} = 3^{k+1} - k - 2$ $\therefore P(k+1)$ is true if $P(k)$ is true. Hence, by mathematical induction, $P(n)$ is true for all positive integers $n \geq 1$	3 marks – correct soluion 2 marks - Proves true for $n = 1$ and incorporates the assumption $P(k)$ into $P(k+1)$ 1 mark - Proves true for $n = 1$
d-i	$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 4(x-2)^{-2} + 8x^{-3}$ $\frac{1}{2}v^2 = -4(x-2)^{-1} - 4x^{-2} + c$ $x = 1, v = 0 \Rightarrow c = 0$ $\therefore \frac{1}{2}v^2 = -4(x-2)^{-1} - 4x^{-2}$ $\therefore v^2 = -\frac{8}{x-2} - \frac{8}{x^2}$	2 marks - Provides correct solution 1 mark - Makes some use of $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$ or equivalent merit
d-ii	 $v^2 \geq 0$ $\Rightarrow -8\left(\frac{1}{x-2} + \frac{1}{x^2}\right) \geq 0$ $\Rightarrow \frac{1}{x-2} + \frac{1}{x^2} \leq 0$ $\Rightarrow \frac{x^2 + x - 2}{x^2(x-2)} \leq 0$ $\Rightarrow \frac{(x+2)(x-1)(x-2)}{x^2(x-2)^2} \leq 0$ From the sketch of the cubic numerator, $x \leq -2$ or $1 \leq x < 2$ [Note that $x - 2$ is originally in the denominator] Since the particle moves continuously, we can only use $1 \leq x < 2$	3 marks – correct solution 2 marks – determine both possible solutions but does not exclude x=2 1 mark – attempts to solve required inequality.

	OR $\frac{x^2 + x - 2}{x^2(x-2)} = 0 \quad x \neq 0 \text{ or } x \neq 2$ $x^2 + x - 2 = 0$ $(x+2)(x-1) = 0$ $x = -2 \text{ or } x = 1$ Test points: when $x < -2$ inequality is true when $-2 < x < 0$ inequality is false when $0 < x < 1$ inequality is flase when $1 < x < 2$ inequality is true when $x > 2$ inequality is false since the particle starts at $x = 1$ and moves continuously only solution is $1 \leq x < 2$ However, students who left $x \leq -2$ were not penalised.	

Question 14

a	$I=\int_0^4 \frac{\sqrt{x}}{\left(x\sqrt{x}+1\right)^2}\,dx$ Let $u^2=x \quad (u\geq 0)$ è $2u\,du=dx$ When $x=0, u=0$ $x=4, u=2$ $\therefore I=\int_0^2 \frac{u}{(u^2.u+1)^2}2u\,du=\int_0^2 \frac{2u^2}{(u^3+1)^2}\,du$ $I=\left[-\frac{2}{3(u^3+1)}\right]_0^2=-\frac{2}{3\times 9}-\left(-\frac{2}{3}\right)=\frac{16}{27}$	3 marks - correct solution 2 marks - Obtains a correct primitive, or equivalent merit 1 mark - Uses an appropriate substitution, including the limits of integration, or equivalent merit
b-i	If x,y are two integers for which one is odd and one is even, then $x+y$ is odd.	1 mark – correct solution
b-ii	Assume x is even, y is odd $\exists\ k,m \in \mathbb{Z}$ such $x=2k$, $y=2m+1$ $x+y=2k+2m+1$ $=2(k+m)+1$ $\therefore\ x+y$ *is odd* $nQ \Rightarrow nP$	2 marks – correct solution 1 mark – attempts correct method ie. makes correct statement $\exists\ k,m \in \mathbb{Z}$ such $x=2k$, $y=2m+1$
c-i	$\frac{1}{a}+\frac{1}{4-a}-1=\frac{4-a+a-a(4-a)}{a(4-a)}$ $=\frac{(a-2)^2}{a(4-a)}\geq 0$ since $0<a<4$ makes the denominator positive Hence $\frac{1}{a}+\frac{1}{4-a}\geq 1$	2 marks – correct solution 1 mark – forms correct inequality
c-ii	Assume $\frac{1}{a}+\frac{1}{4-b}<1, \frac{1}{b}+\frac{1}{4-c}<1$ and $\frac{1}{c}+\frac{1}{4-a}<1$ By addition, $\left(\frac{1}{a}+\frac{1}{4-b}\right)+\left(\frac{1}{b}+\frac{1}{4-c}\right)+\left(\frac{1}{c}+\frac{1}{4-a}\right)<3$ $\Rightarrow\left(\frac{1}{a}+\frac{1}{4-a}\right)+\left(\frac{1}{b}+\frac{1}{4-b}\right)+\left(\frac{1}{c}+\frac{1}{4-c}\right)<3$ But, from part (i), each bracket is at least 1, a contradiction. Hence, at least one of the three numbers is at least 1	3 marks – correct solution 2 marks - Makes significant progress towards a solution. 1 mark - Makes little progress towards a solution

d	Option 1 $I_n = \int_0^1 \frac{x^{2n}}{\sqrt{1+x^2}} dx$ $I_n = \int_0^1 \frac{x^2 x^{2n-2}}{\sqrt{1+x^2}} dx$ since $n \geq 2$ $I_n = \int_0^1 \frac{(1+x^2-1)x^{2n-2}}{\sqrt{1+x^2}} dx$ $I_n = \int_0^1 \frac{(1+x^2)x^{2n-2}}{\sqrt{1+x^2}} dx - \int_0^1 \frac{x^{2(n-1)}}{\sqrt{1+x^2}} dx$ $I_n = \int_0^1 x^{2n-2}\sqrt{1+x^2} dx - I_{n-1}$ For $\int_0^1 x^{2n-2}\sqrt{1+x^2} dx$, integrate by parts as follows: $u = \sqrt{1+x^2} \quad \therefore \frac{du}{dx} = \frac{x}{\sqrt{1+x^2}}$ $\frac{dv}{dx} = x^{2n-2} \quad \therefore v = \frac{1}{2n-1}x^{2n-1}$ $\therefore I_n = \left[\frac{x^{2n-1}\sqrt{1+x^2}}{2n-1}\right]_0^1 - \frac{1}{2n-1}\int_0^1 x^{2n-1}\frac{x}{\sqrt{1+x^2}} dx - I_{n-1}$ $I_n = \frac{\sqrt{2}}{2n-1} - \frac{1}{2n-1}\int_0^1 \frac{x^{2n}}{\sqrt{1+x^2}} dx - I_{n-1}$ $I_n = \frac{\sqrt{2}}{2n-1} - \frac{1}{2n-1}I_n - I_{n-1}$ $(2n-1)I_n = \sqrt{2} - I_n - (2n-1)I_{n-1}$ $\therefore 2n\, I_n + (2n-1)I_{n-1} = \sqrt{2}$ as required	4 marks – correct solution. 3 marks - Makes substantial progress, identifying the terms I_n and I_{n-1} 2 marks - Attempts to use integration by parts and attempts a manipulation to obtain I_{n-1} or I_n, or equivalent merit 1 mark - Attempts to use integration by parts, or equivalent merit

d.

Option 2

$$I_n = \int_0^1 \frac{x^{2n}}{\sqrt{1+x^2}}\,dx$$

$$= \int_0^1 \frac{x.x^{2n-1}}{\sqrt{1+x^2}}dx$$

$$u = x^{2n-1} \qquad dv = \frac{x}{\sqrt{1+x^2}}$$

$$du - (2n-1)x^{2n-2} \qquad v = \int \frac{x}{\sqrt{1+x^2}}\,dx$$

$$= \frac{1}{2}\int \frac{2x}{\sqrt{1+x^2}}\,dx$$

$$= (1+x^2)^{\frac{1}{2}} = \sqrt{1+x^2})$$

$$I_n = \left[x^{2n-1}\sqrt{1+x^2}\,\right]_0^1 - \int_0^1 \sqrt{1+x^2}.\,(2n-1).x^{2n-2}\,dx$$

$$= \sqrt{2} - \int_0^1 \sqrt{1+x^2}.\,(2n-1).x^{2n-2}\,dx$$

$$= \sqrt{2} - (2n-1).\int_0^1 \sqrt{1+x^2}.\,x^{2n-2}\,dx$$

$$= \sqrt{2} - (2n-1).\int_0^1 \frac{(1+x^2).\,x^{2n-2}}{\sqrt{1+x^2}}\,dx$$

$$= \sqrt{2} - (2n-1)\left[\int_0^1 \frac{x^{2n-2}}{\sqrt{1+x^2}}\,dx + \int_0^1 \frac{x^2}{\sqrt{1+x^2}}\,dx\right]$$

$$= \sqrt{2} - (2n-1)[I_{n-1} + I_n]$$

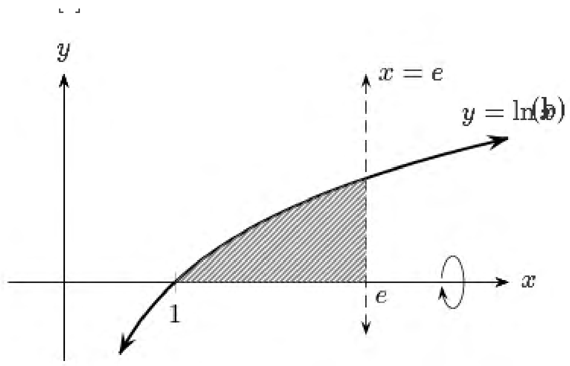
$$= \sqrt{2} - (2n-1)I_{n-1} - (2n-1)I_n]$$

$$I_n + 2nI_n - I_n + (2n-1)I_{n-1} = \sqrt{2}$$

$$2nI_n + (2n-1)I_{n-1} = \sqrt{2}$$

Question 15

<table>
<tr>
<td>a</td>
<td>



$$V = \pi \int_1^e y^2 \, dx$$
$$= \pi \int_1^e (\ln x)^2 \, dx$$

Finding the integral by parts/inserting 'phantom' term

$$u = (\ln x)^2 \qquad dv = 1$$
$$du = 2 \ln x \times \frac{1}{x} \qquad v = x$$
$$= \frac{2}{x} \ln x$$

$$V = \pi \left(\left[x (\ln x)^2 \right]_1^e - \int_1^e \frac{2}{\cancel{x}} \ln x \times \cancel{x} \, dx \right)$$
$$= \pi \left(e - 2 \int_1^e \ln x \, dx \right)$$

Use the area about the y axis:

$$\int_1^e \ln x \, dx = (e \times 1) - \int_{y=0}^{y=1} e^y \, dy$$
$$= e - [e - 1]$$
$$= 1$$
$$\therefore V = \pi(e - 2) \text{ units}^3$$

</td>
<td>
3 marks – correct solution

2 marks – correct use of IBP

1 mark
</td>
</tr>
</table>

b-i	Let $\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$, $\left\lvert \underset{\sim}{r} - \left(3\underset{\sim}{i} + \underset{\sim}{j} + 4\underset{\sim}{k}\right)\right\rvert = \sqrt{35}$ $\left\lvert (x-3)\underset{\sim}{i} + (y-1)\underset{\sim}{j} + z - 4\underset{\sim}{k}\right\rvert = \sqrt{35}$ $\therefore (x-3)^2 + (y-1)^2 + (z-4)^2 = 35$	1 mark
b-i	$\underset{\sim}{r} = \begin{pmatrix}2\\0\\3\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\-1\end{pmatrix}$ If the line is a tangent, then only one unique value of λ exists for the expression $\left\lvert \underset{\sim}{r} - \begin{pmatrix}3\\1\\4\end{pmatrix}\right\rvert = \sqrt{35}$ Test for values of λ that satisfy following $\left\lvert r - \begin{pmatrix}3\\1\\4\end{pmatrix}\right\rvert = \sqrt{35}$ LHS $= \left\lvert \begin{pmatrix}2\\0\\3\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\-1\end{pmatrix} - \begin{pmatrix}3\\1\\4\end{pmatrix}\right\rvert$ $= \left\lvert \begin{pmatrix}-1+\lambda\\-1+2\lambda\\-1-\lambda\end{pmatrix}\right\rvert$ $= \sqrt{(\lambda-1)^2 + (2\lambda-1)^2 + (-1-\lambda)^2}$ $= \sqrt{\lambda^2 - 2\lambda + 1 + 4\lambda^2 - 4\lambda + 1 + \lambda^2 + 2\lambda + 1}$ $= \sqrt{6\lambda^2 - 4\lambda + 3} = \sqrt{35}$ $6\lambda^2 - 4\lambda + 3 = 35$ $6\lambda^2 - 4\lambda - 32 = 0$ $3\lambda^2 - 2\lambda - 16 = 0$ $\Delta = (-2)^2 - 4(-16)(3)$ $= 196 > 0$ $\therefore$ 2 solutions $\Rightarrow$ not a tangent	3 marks

c	Show z, w exist given $\frac{1}{z}+\frac{1}{w}=\frac{1}{z+w}$ $\therefore$ $$\frac{1}{z}+\frac{1}{w}=\frac{1}{z+w}$$ $$\frac{z+w}{z}+\frac{z+w}{w}=1$$ $$\frac{z}{z}+\frac{w}{z}+\frac{z}{w}+\frac{w}{w}=1$$ $$1+\frac{z}{w}+\frac{w}{z}+1=1$$ $$\frac{z}{w}+\frac{w}{z}+1=0 \quad \therefore \text{ Line 1}$$ Let $k = \left(\frac{z}{w}\right)$ Line 1 becomes $$k+\frac{1}{k}+1=o$$ $$k^2+1+k=0$$ $$k^2+k+1=0$$ $$k^2+k=1=0$$ $$k^2+k+\frac{1}{4}+\frac{3}{4}=0$$ $$\left(k+\frac{1}{2}\right)^2+\frac{3}{4}=0$$ $$\left(k+\frac{1}{2}\right)^2=-\frac{3}{4}$$ $$k=-\frac{1}{2}\pm\frac{\sqrt{3}\,i}{2}$$ $\therefore$ as k exists as a complex number then $\frac{z}{w}$ exists as complex number hence complex numbers z and w exist s such that $\frac{1}{z}+\frac{1}{w}=\frac{1}{z+w}$ eg $z =$ $-1+3i$ and $w = 2$	3 marks – correct solution 2 marks – simplifies the question by assigning a value for either z or w 1 mark – correct solution for assumption that $\|z\| = \|w\| = 1$

d-i	$a^2+b^2+c^2 \geq ab+ac+bc$ $(a-b)^2 \geq 0$ $a^2-2ab+b^2 \geq 0$ $a^2+b^2 \geq 2ab$ Similarly $a^2+c^2 \geq 2ac$ $b^2+c^2 \geq 2bc$ $2a^2+2b^2+2c^2 \geq 2ab+2ac+2bc$ $a^2+b^2+c^2 \geq ab+ac+bc$	1 mark
e-i	$(a+b+c)^2 \geq 3(ab+ac+bc)$ LHS $= (a+b+c)^2$ $= a^2+b^2+c^2+2ab+2bc+2ac$ $= a^2+b^2+c^2+2ab+2bc+2ac$ $\geq ab+bc+ac+2ab+2bc+2ac$ $\geq 3ab+3bc+3ac$ $\geq 3(ab+bc+ac)$	1 mark

e-ii	***hence prove:*** $\boldsymbol{a^2+b^2+c^2 \geq 3\sqrt[3]{a^2b^2c^2}}$ $(a-b)^2 \geq 0$ $a^2 - 2ab + b \geq 0$ $a^2 - ab + b^2 \geq ab$ $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ $a^3 + b^3 \geq (a+b)ab \quad \ldots(1)$ *Similarly* $b^3 + c^3 \geq (c+b)cb \ldots(2)$ $a^3 + c^3 \geq (a+c)ac \quad \ldots(3)$ *Multiply Lines 1, 2, 3 by* $\frac{c}{c}, \frac{a}{a}$ *and* $\frac{b}{b}$ *gives* $a^3 + b^3 \geq \left(\frac{a}{c} + \frac{b}{c}\right)abc$ $b^3 + c^3 \geq \left(\frac{b}{a} + \frac{c}{a}\right)abc$ $a^3 + c^3 \geq \left(\frac{a}{b} + \frac{c}{b}\right)abc$ *Now add these new inequalities* $2(a^3 + b^3 + c^3) \geq abc\left(\frac{a}{c} + \frac{b}{c} + \frac{b}{a} + \frac{c}{a} + \frac{a}{b} + \frac{c}{b}\right)$ $2(a^3 + b^3 + c^3) \geq abc\left(\frac{a}{b} + \frac{b}{a} + \frac{a}{c} + \frac{c}{a} + \frac{b}{c} + \frac{c}{b}\right)$ $2(a^3 + b^3 + c^3) \geq abc\left(\frac{a^2+b^2}{ab} + \frac{a^2+c^2}{ac} + \frac{b^2+c^2}{bc}\right)$ $nb \; a^2 + b^2 \geq 2ab$ $\therefore \; 2(a^3 + b^3 + c) \geq abc\left(\frac{2ab}{ab} + \frac{2ac}{ac} + \frac{2bc}{bc}\right)$ $2(a^3 + b^3 + c^3) \geq 6abc$ $a^3 + b^3 + c^3 \geq 3abc$ $Let\ a \to \sqrt[3]{a} \quad b \to \sqrt[3]{b} \quad c \to \sqrt[3]{c}$ $a + b + c \geq 3\sqrt[3]{abc}$ $Let\ a \to \sqrt[3]{a^2} \quad b \to \sqrt[3]{b^2} \quad c \to \sqrt[3]{c^2}$ $a^2 + b^2 + c^2 \geq 3\sqrt[3]{a^2b^2c^2}$	3 marks - correct 2 marks – correct however used AM v GM (Note: if "Hence" had not been used in the question – it would have been legitimate to use AMvGM statement. 1 mark – some progress using part i and ii

Question 16

<table>
<tr><td>a-i</td><td>Clearly, $z = -1 = cis\,\pi$ is a solution.

The solutions lie on the unit circle, equally spaced.

Hence, solutions are:

$z = cis\,\pi,\ \ cis\frac{\pi}{3},\ \ cis\left(-\frac{\pi}{3}\right)$</td><td>2 marks – correct solution

1 mark - Writes one of the solutions in modulus-argument form</td></tr>
<tr><td>a-ii</td><td>Using part (ii),
$z^2 + 2 = cis\,\pi,\ \ cis\frac{\pi}{3},\ \ cis\left(-\frac{\pi}{3}\right)$

$z^2 + 2 = cis\,\pi$:
$z^2 + 2 = -1$
$z^2 = -3$
$z = \pm i\sqrt{3}$

$z^2 + 2 = cis\frac{\pi}{3}$:
$z^2 + 2 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$
$z^2 = -\frac{3}{2} + \frac{\sqrt{3}}{2}i$
$z^2 = \sqrt{3}\,cis\frac{5\pi}{6}$
$z = \sqrt[4]{3}\,cis\frac{5\pi}{12},\ \ \sqrt[4]{3}\,cis\left(\frac{5\pi}{12} - \pi\right)$
$z = \sqrt[4]{3}\,cis\frac{5\pi}{12},\ \ \sqrt[4]{3}\,cis\left(-\frac{7\pi}{12}\right)$

$z^2 + 2 = cis\left(-\frac{\pi}{3}\right)$:
$z^2 + 2 = \frac{1}{2} - \frac{\sqrt{3}}{2}i$
$z^2 = -\frac{3}{2} - \frac{\sqrt{3}}{2}i$
$z^2 = \sqrt{3}\,cis\left(-\frac{5\pi}{6}\right)$
$z = \sqrt[4]{3}\,cis\left(-\frac{5\pi}{12}\right),\ \ \sqrt[4]{3}\,cis\left(-\frac{5\pi}{12} + \pi\right)$
$z = \sqrt[4]{3}\,cis\left(-\frac{5\pi}{12}\right),\ \ \sqrt[4]{3}\,cis\left(\frac{7\pi}{12}\right)$

∴ The six solutions are:
$z = \pm i\sqrt{3},\ \ \sqrt[4]{3}\,cis\frac{5\pi}{12},\ \ \sqrt[4]{3}\,cis\left(-\frac{5\pi}{12}\right),\ \ \sqrt[4]{3}\,cis\frac{7\pi}{12},\ \ \sqrt[4]{3}\,cis\left(-\frac{7\pi}{12}\right)$</td><td>3 marks – correct solution

2 marks - Finds two solutions, each from a different substitution of the 3 expressions from (i), or equivalent merit. Equivalent forms of solutions were accepted.

1 mark - Finds one solution, or equivalent merit</td></tr>
</table>

b-i	Find $\overrightarrow{PR}$ and $\overrightarrow{QS}$ in terms of e. $\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP}$ $\overrightarrow{OR} = \dfrac{\overrightarrow{OE} + \overrightarrow{OC}}{2} = \dfrac{4e\underline{k} + 4\underline{i} + 4\underline{j}}{2} = 2\underline{i} + 2\underline{j} + 2e\underline{k}$ $\overrightarrow{OP} = \dfrac{\overrightarrow{OA} + \overrightarrow{OB}}{2} = -4\underline{j}$ $\therefore \overrightarrow{PR} = 2\underline{i} + 6\underline{j} + 2e\underline{k}$ Similarly, $\overrightarrow{QS} = -6\underline{i} - 2\underline{j} + 2e\underline{k}$	2 Marks: Provides correct solution. (Must include working for QS) 1 Mark: Finds $\overrightarrow{PR}$ or $\overrightarrow{QS}$, or equivalent merit
b-ii	For Line l, use point P and direction vector $\overrightarrow{PR}$: $\therefore$ Line l: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 6 \\ 2e \end{pmatrix}$ For Line s, use point Q and direction vector $\overrightarrow{QS}$: $\therefore$ Line s: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -6 \\ -2 \\ 2e \end{pmatrix}$	2 Marks: Provides correct solution. OR Obtains equations consistent with an error in part (i) 1Mark: Writes the vector equation of either Line l or Line s, or equivalent merit OR Obtains the correct direction vector for **each** line.
b-iii	To find the point of intersection, equate the x components: $2\lambda = 4 - 6\mu$ $\therefore \lambda = 2 - 3\mu$ [1] Equate the y components and use [1]: $-4 + 6\lambda = -2\mu$ $-4 + 6(2 - 3\mu) = -2\mu$ $\therefore \mu = \frac{1}{2}$ and $\lambda = \frac{1}{2}$ $\therefore$ Point of intersection is $(1, -1, e)$ $\therefore \overrightarrow{OM} = \underline{i} - \underline{j} + e\underline{k}$	2 Marks - Uses the values of the parameters to find the point of intersection, making one error 1 Mark - Correctly equates x and y components and attempts to solves simultaneously to find the values of the parameters
b-iv	$\overrightarrow{EB} = \overrightarrow{OB} - \overrightarrow{OE} = \underline{4i} - 4\underline{j} - 4e\underline{k}$ If $\overrightarrow{OM} \perp \overrightarrow{EB}$, $\overrightarrow{OM} \cdot \overrightarrow{EB} = 0$:	3 Marks: Correct answer 2 Marks: Obtains two correct values for e, $cos\Theta$ or Θ

		$\begin{pmatrix}1\\-1\\e\end{pmatrix}\cdot\begin{pmatrix}4\\-4\\-4e\end{pmatrix}=0$ $4+4-4e^2=0$ $e=\sqrt{2}\quad(e>0)$ The angle between $\overrightarrow{PR}$ and $\overrightarrow{QS}$: $\overrightarrow{PR}=2\underline{i}+6\underline{j}+2\sqrt{2}\,\underline{k}$ $\overrightarrow{QS}=-6\underline{i}-2\underline{j}+2\sqrt{2}\,\underline{k}$ $\therefore \cos\theta=\dfrac{\overrightarrow{PR}\cdot\overrightarrow{QS}}{\lvert\overrightarrow{PR}\rvert\lvert\overrightarrow{QS}\rvert}=\dfrac{-12-12+8}{\left(\sqrt{2^2+6^2+\left(2\sqrt{2}\right)^2}\right)^2}=-\dfrac{1}{3}$ $\therefore \theta\cong 109^\circ$ (nearest degree)	OR Obtains values of *cosΘ* and Θ consistent with an incorrect value of *e.* 1 Mark: Finds a correct expression for vector EB OR Finds the value of *e* OR Finds an angle consistent with incorrect values for EB and *e*.